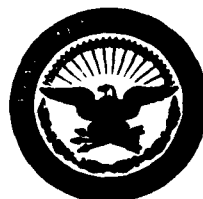


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Terrestrial Spectroscopy

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Rehovot, Israel.

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ABSTRACT: It is shown that the rotation of the earth splits each line of its spectrum of order n into a multiplet of $(2n+1)$ lines. The theory is developed for multiplet intensities in the case of a point-source. The results are in good agreement with the multiplets observed gravimetrically and seismically.

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Terrestrial Spectroscopy

by

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The discovery of the Zeeman effect in 1896 came at a time when the basic concepts needed for its interpretation had been worked out by Lorentz⁽¹⁾. Already in his first paper⁽²⁾ Zeeman reports a verification of the polarization of the lines, an effect which Lorentz had advised him to look for. He gives a derivation, on the basis of the Lorentz theory, of the formula for the splitting of the original frequency σ_0 by a magnetic field H :

$$\sigma = \sigma_0 \pm \frac{eH}{2mc} \quad (1)$$

This relation had been derived independently by Larmor. Larmor proved⁽³⁾ that the effect of a magnetic field on the orbit of a charged particle is analogous to the effect of rotation on the frequency of a mechanical system. While the Zeeman effect provides ample verification of the electromagnetic part of Larmor's analogy, an experimental demonstration of the mechanical counterpart has been wanting. A striking case of the splitting of the natural frequency of a purely mechanical system by rotation presented itself recently when the records of the great Chilean earthquake of May 22, 1960 were analyzed. This earthquake excited the natural oscillations of the earth. The gravest modes $n=2$ (53.7 min) and $n=3$ (35.5 min) appeared as doublets in the spectra of both the gravimetric records⁽⁴⁾ and the strain-meter records⁽⁵⁾.

The free oscillations of the earth are governed by gravitational and elastic forces. Taking the distribution of density $\rho(r)$, and of the elastic constants $\lambda(r)$ and $\mu(r)$, as inferred primarily from seismic as well as from other geophysical data, we have determined the terrestrial spectrum for several proposed models of the earth^{(6),(7),(8),(9)}. The periods deduced from spectral analysis of the strain-meter records (52 lines) agreed with the gravimetric values (49 lines) to within 1%; and this was also the measure of agreement with the theoretical spectrum⁽⁹⁾ for the Gutenberg earth model, and to a lesser extent with the spectrum for the Bullen B model. Free modes ranging from spherical harmonic order $n=2$ up to $n=38$ were identified seismically, and up to $n=41$ gravimetrically.

We have investigated the effect of rotation on the spectrum of the earth by carrying out a first-order perturbation calculation in the parameter $\alpha = \omega/\sigma_0$, where ω denotes the angular rotation of the earth. In the absence of rotation, the components of displacement u_0 , v_0 and w_0 in a spherical system of coordinates (r, θ, ϕ) are given by

$$u_0 = U_0(r)Y_{nm}, \quad v_0 = V_0(r)\frac{\partial}{\partial \theta} Y_{nm}, \quad (2)$$

$$w_0 = \frac{V_0(r)}{\sin \theta} \frac{\partial Y_{nm}}{\partial \phi}, \quad Y_{nm}(\theta, \phi) = P_n^m(\cos \theta)e^{im\phi}, \quad (3)$$

where a factor $e^{i\sigma t}$ has been omitted. The characteristic frequencies $\sigma_0(n)$ are degenerate, and do not depend on the azimuthal number m . The introduction of a slow rotation ω ($\ll \sigma_0$) removes the degeneracy, each line $\sigma_0(n)$ being split into $(2n+1)$ lines σ_n^m given by

$$\sigma_n^m = \sigma_0(n) + m\tau(n)\omega \quad -n \leq m \leq n \quad (4)$$

with

$$\tau(n) = \frac{\int_0^a \rho r^2 (2U_0 V_0 + V_0^2) dr}{\int_0^a \rho r^2 [U_0^2 + n(n+1)V_0^2] dr}, \quad (5)$$

in the case of spheroidal oscillations, and

$$\tau(n) = \frac{1}{n(n+1)} \quad (6)$$

in the case of torsional oscillations.

Table 1 gives the periods in the multiplets for the oscillations of spherical harmonic order $n=2$ and $n=3$ of model Bullen B. These are based on the previously determined values⁽⁷⁾ of $\sigma_0(n)$, and on equations (4) and (5). Also shown are theoretical intensities of the gravimetric and strain-meter lines for an observing station at Los Angeles ($\theta = 56^\circ$). These were determined on the assumption of an explosive compressional point-source at the earthquake focus in Chile ($\theta_0 = 128^\circ$).

Table 1
Theoretical and observed multiplets for Los Angeles. ($\theta = 56^\circ$)

n	m	T _{min}	Amplitude		Observed periods	
			Gravimetric	Strain	Gravimetric ⁽⁴⁾	Strain ⁽⁵⁾
2	- 2	55.33	.115	.302	54.98	54.7
2	- 1	54.50	.242	.615		
2	0	53.70	.0015	.0023		
2	1	52.92	.242	.615	52.80	53.1
2	2	52.15	.115	.302		
3	- 3	35.99	.396	.401	35.87	35.9
3	- 2	35.83	1.257	1.46		
3	- 1	35.67	.282	.377		
3	0	35.50	.624	.873		
3	1	35.34	.282	.377		
3	2	35.18	1.257	1.46	35.24	35.2
3	3	35.03	.396	.401		

As for the periods, the feature of interest is the observed interval between the doublets rather than the central frequency, since the latter depends on the particular earth model assumed. We note the following:

1. Gravimetric. In the case $n=2$, the central line $m=0$ should be very weak, and was actually not observed. The theoretical interval between the strongest $m=-1$ and $m=1$ lines is 1.6 minutes, as against the observed interval of 2.2 minutes. Since, however, the $m=\pm 2$ lines are theoretically half as strong as the $m=\pm 1$ lines, the observed lines may represent the unresolved pairs $m=1$ and $m=2$. The average periods of the pairs, weighted according to their amplitudes, are 54.77 and 52.67 minutes, giving an interval of 2.1 minutes, which is close to the observed interval of 2.2 minutes.

In the case $n=3$, the strongest lines should be the pair $m=\pm 2$, which are very close to the observed two lines. The $m=0$ line should be half as strong, and should have been observed if the noise level were low enough and the resolution adequate.

In addition to the doublets in the fundamental $n=2$ and $n=3$ modes, the first overtone of the $n=3$ mode was also reported⁽⁴⁾ as a gravitational doublet: $T = 17.88$ and 17.68 minutes. Theoretically, the relative amplitudes within a multiplet for the overtones should be the same as for the fundamental, as given in Table 1. The strongest lines in all the overtones of $n=3$ should therefore be again the pair $m=\pm 2$. The period-interval for the pair comes out 0.19 minutes, in agreement with the observed value of 0.2 minutes.

2. Seismic. Two doublets were reported by Benioff, Press and Smith⁽³⁾ from a Fourier analysis of the strain seismogram at Isabella, California. In Figure 1 is reproduced their spectral intensity curve in the vicinity of the $n=2$ pair of 54.7 and 53.1 minutes. In the same figure are shown, by arrows, the theoretical separations and amplitudes of the five lines making up the $n=2$ quintet, in accordance with the results given in Table 1. The central $m=0$ line should be extremely weak, and is indeed below the background noise. The $m=2$ line is not far from the observed intensity level. On the other hand, the $m=-2$ line is definitely above the observed intensity in its vicinity. One would expect a considerable rise in intensity immediately to the left of this line, which is outside the limit of the figure.

In Figure 2 is shown the observed spectral curve for $n=3$, and the theoretical positions and amplitudes of the $n=3$ septet. The central $m=0$ line is defined, and its theoretical intensity is close to the observed value. The $m=\pm 1$ and $m=\pm 3$ pairs are close to the general noise level in their respective neighborhoods.

It is seen that, in general, the doublets observed are those lines of the multiplet which are strongest theoretically, and that the missing lines would not be expected to stand out above the observed noise level.

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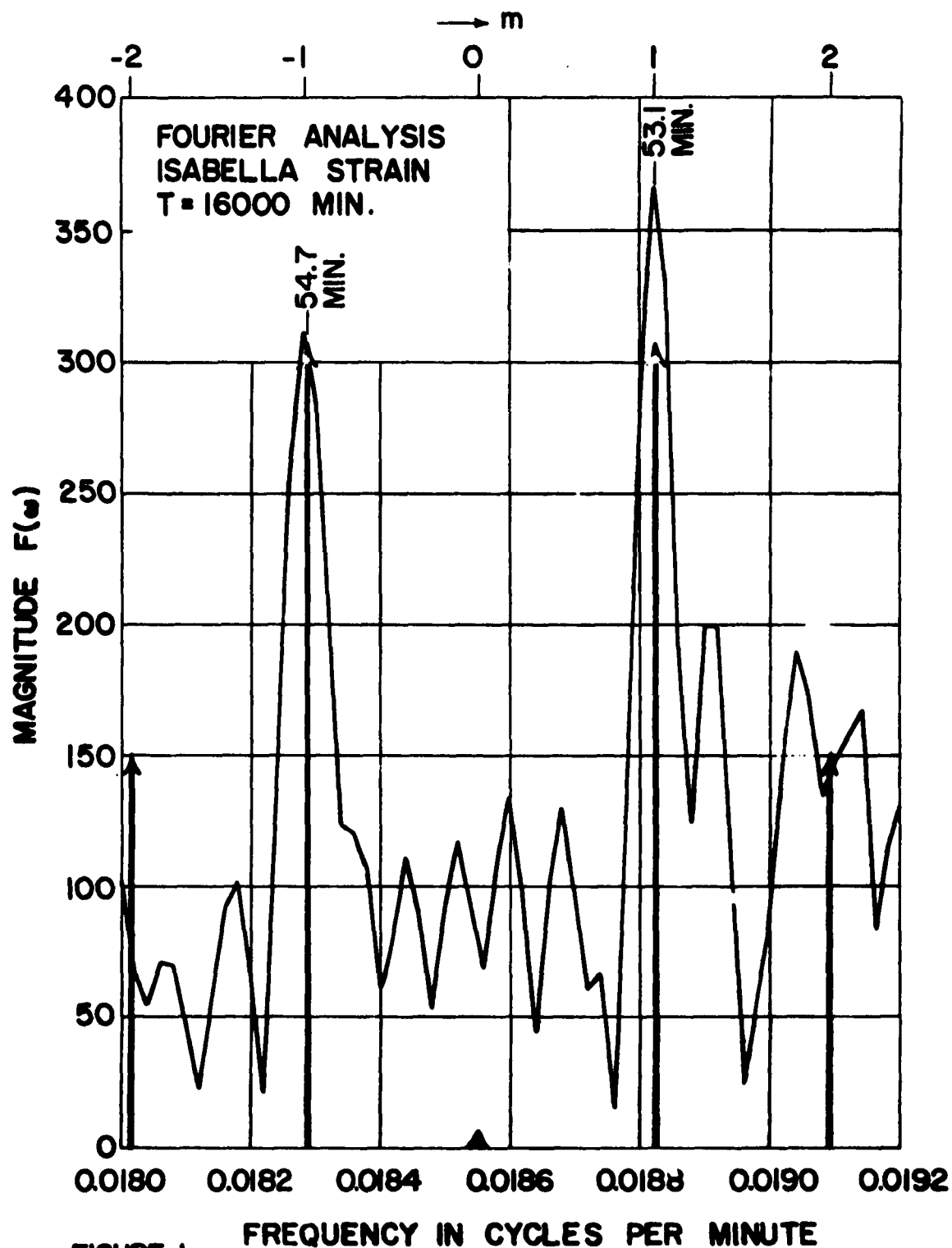


FIGURE 1.

$n=2$. Arrows show positions and amplitudes of multiplet. — Spectral intensity of Chilean earthquake observed at Isabella by Benloff, Press and Smith.

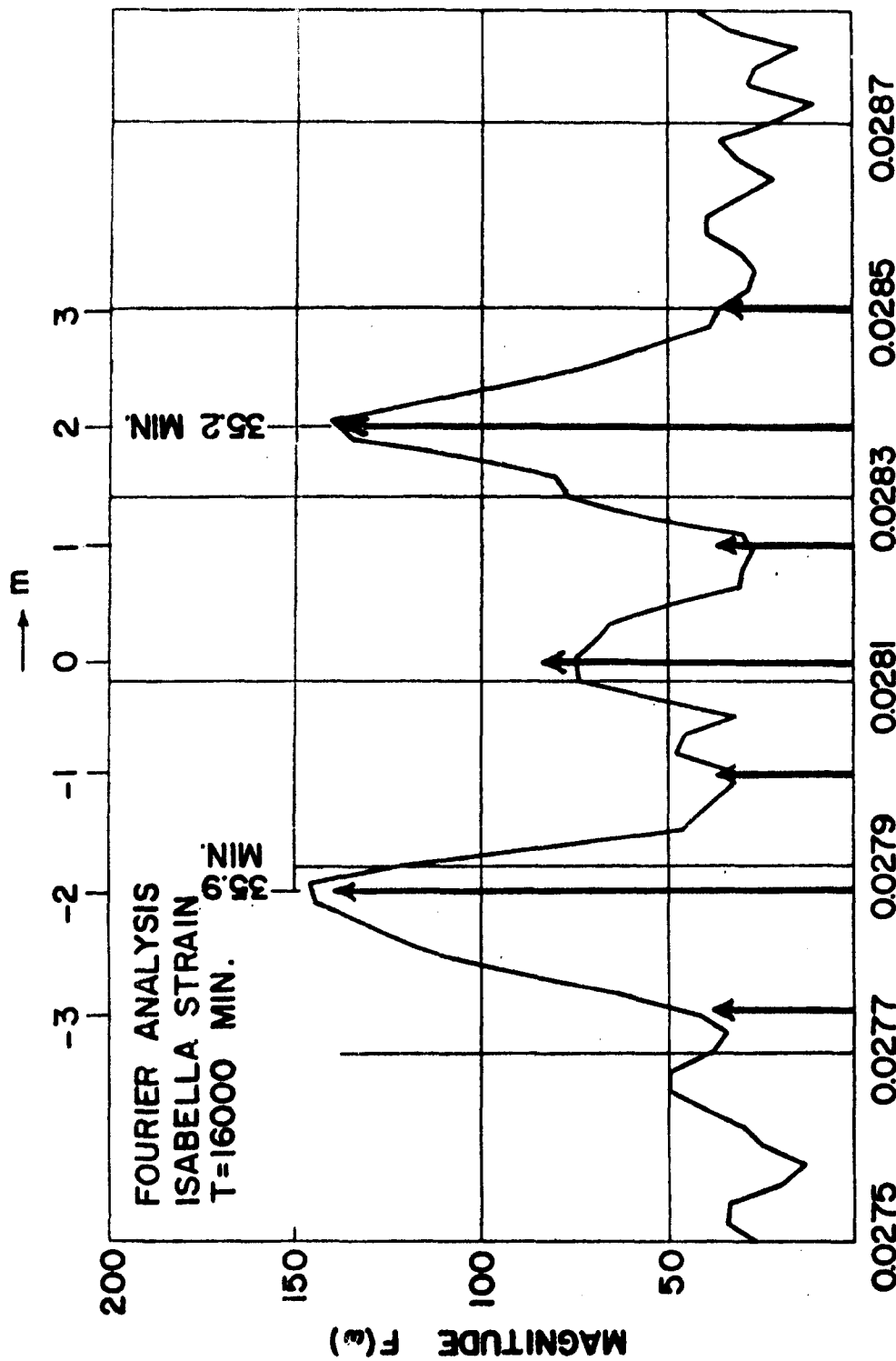


FIGURE 2.

$n=3$. Arrows show positions and amplitudes of multiplet. — Spectral intensity of Chilean earthquake observed at Isabella by Benioff, Press and Smith.

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